# Section 1.10 Modeling and Variation

The table shows the population $y$ (in millions) of the United States from 2008 through 2015.

| Year | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
|---|---|---|---|---|---|---|---|---|
| Population, $y$ | 304.1 | 306.8 | 309.3 | 311.7 | 314.1 | 316.5 | 318.9 | 321.2 |

A linear model that approximates the data is:

$y = 2.43t + 284.9, \ 8 \le t \le 15$

How well does the data fit? Check on graphing calculator

## Direct Variation

The following statements are equivalent.

1. $y$ **varies directly** as $x$.
2. $y$ is **directly proportional** to $x$.
3. $y = \boldsymbol{k}x$ for some nonzero constant $k$.

$\boldsymbol{k}$ is the **constant of variation** or the **constant of proportionality**

Find a direct variation model that relates $y$ and $x$.

x = 5 and y = 12

Simplest direct variation is a line with a $y$-intercept equal to zero.

$$y = mx + b,\ b \neq 0$$

or

$$y = kx$$

In Pennsylvania, the state income tax is directly proportional to *gross income*. You work in Pennsylvania and your state income tax deduction is \$46.05 for a gross monthly income of \$1500. Find a mathematical model that gives the Pennsylvania state income tax in terms of gross income.

## Direct Variation as an nth Power

The following statements are equivalent.

1. $y$ **varies directly** as the nth power of $x$.
2. $y$ is **directly proportional** to the nth power of $x$.
3. $y = \boldsymbol{k}x^n$ for some nonzero constant $k$.

$\boldsymbol{k}$ is the **constant of variation** or the **constant of proportionality**

$A = \pi r^2$ is an example of a direct variation to the nth power.

The area A is directly proportional to the square of the radius r. $\pi$ is the constant of proportionality.

The distance a ball rolls down an inclined plane is directly proportional to the square of the time it rolls. During the first second, the ball rolls 8 feet.

a) Write an equation relating the distance traveled to the time.

b) How far does the ball roll during the first 3 seconds?

## Inverse Variation

The following statements are equivalent.

1. $y$ **varies inversely** as the nth power of $x$.
2. $y$ is **inversely proportional** to the nth power of $x$.
3. $y = \boldsymbol{k/x}$ for some nonzero constant $k$.

***k*** is the **constant of variation** or the **constant of proportionality**

A company has found that the demand for one of its products varies inversely as the price of the product. When the price is $6.25, the demand is 400 units. Approximate the demand when the price is $5.75.

A company has found that the demand for one of its products varies inversely as the price of the product. When the price is $2.75, the demand is 600 units. Approximate the demand when the price is $3.25.

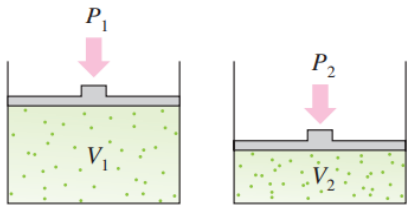
## Combined Variation

Some applications of variation involve problems with ***both*** direct and inverse variations in the same model.

A gas law states that the volume of an enclosed gas varies inversely as the pressure and directly as the temperature. The pressure of a gas is 0.75 kilogram per square centimeter when the temperature is 294 K and the volume is 8000 cubic centimeters.

a) Write the equation relating pressure, temperature, and volume.

b) Find the pressure when the temperature is 300 K and the volume is 7000 cubic centimeters.




$P_2 > P_1$ then $V_2 < V_1$

If the temperature is held constant and pressure increases, then the volume decreases.

## Joint Variation

The following statements are equivalent.

1. $z$ **varies jointly** as $x$ and $y$.
2. z is **jointly proportional** to $x$ and $y$.
3. $z = \boldsymbol{k}xy$ for some nonzero constant $k$.

$\boldsymbol{k}$ is the **constant of variation** or the **constant of proportionality**

The simple interest for an investment is jointly proportional to the time and the principal. After one quarter (3 months), the interst on a principal of $5000 is $43.75.

a) Write an equation relating the interest, principal, and time.

b) Find the interest after 3 quarters.

The kinetic energy E of an object varies jointly with the object's mass *m* and the square of the object's velocity *v*. An object with a mass of 50 kilograms traveling at 16 meters per second has a kinetic energy of 6400 joules. What is the kinetic energy of an object with a mass of 70 kilograms traveling at 20 meters per second?

Determine whether the variation model represented by the ordered pairs $(x, y)$ is of the form $y = kx$ or $y = k/x$, and find $k$. Then write a model that relates $y$ and $x$.

(5, 2), (10, 4), (15, 6), (20, 8), (25, 10)

(5, 24), (10, 12), (15, 8), (20, 6), (25, 24/5)

Section 1.10

Assignment:

Pgs. 100-103: #10, 17, 19-43 odd, 54-57, 62, 67, 75-78