Precalculus

6.3-6.5 Review – vectors

Name Key

Period_____ Date_____

Determine whether u and v are equivalent. Explain.

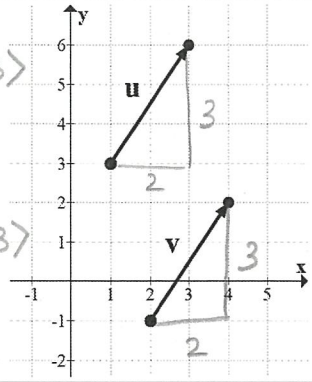
1.

$\|u\| = \sqrt{2^2+3^2}$ $\langle 2,3\rangle$

$\|u\| = \sqrt{13}$

$\|v\| = \sqrt{2^2+3^2}$ $\langle 2,3\rangle$

$\|v\| = \sqrt{13}$



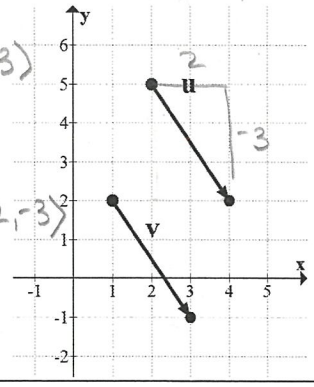
2.

$\|u\| = \sqrt{2^2+3^2}$ $\langle 2,-3\rangle$

$\|u\| = \sqrt{13}$

$\|v\| = \sqrt{2^2+3^2} = \langle 2,-3\rangle$

$\|v\| = \sqrt{13}$



Find the component form and magnitude of the vector v.

3.

Initial point: $(0,-3)$

Terminal point: $(4,5)$

$V = \langle 4,8\rangle$

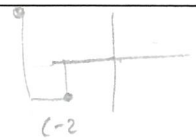
standard $V = 4i + 8j$

$\|V\| = \sqrt{80} = 4\sqrt{5}$

4.

Initial point: $(-2,-3)$

Terminal point: $(-6,5)$



$V = \langle -4, 8\rangle$

Standard $V = -4i + 8j$

$\|V\| = \sqrt{80} = 4\sqrt{5}$

Find: a) u + v b) −2u − v both algebriically and graphically. Show the resultant vector.

5. $\mathbf{u} = \langle 4,5\rangle$

$\mathbf{v} = \langle 0,-3\rangle$

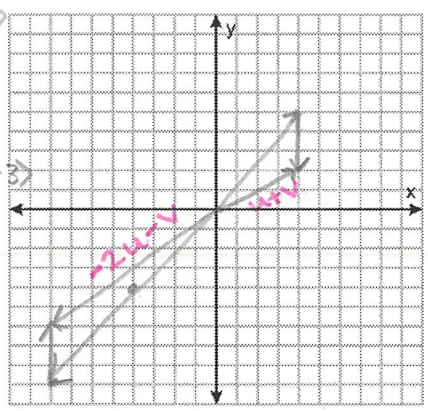
a) $\langle 4,5\rangle + \langle 0,-3\rangle$

$\langle 4,2\rangle$

b) $-2\langle 4,5\rangle - \langle 0,-3\rangle$

$\langle -8,-10\rangle + \langle 0,3\rangle$

$\langle -8,-7\rangle$



6. $\mathbf{u} = \langle 1,-3\rangle$

$\mathbf{v} = \langle 3,-2\rangle$

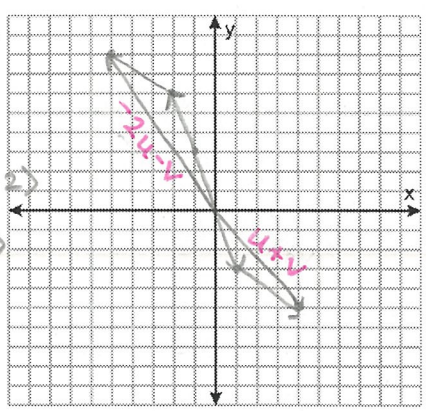
a) $\langle 1,-3\rangle + \langle 3,-2\rangle$

$\langle 4,-5\rangle$

b) $-2\langle 1,-3\rangle - \langle 3,-2\rangle$

$\langle -2,6\rangle + \langle -3+2\rangle$

$\langle -5,8\rangle$



7. $\mathbf{u} = -7\mathbf{i} - 3\mathbf{j}$

$\mathbf{v} = 4\mathbf{i} - \mathbf{j}$

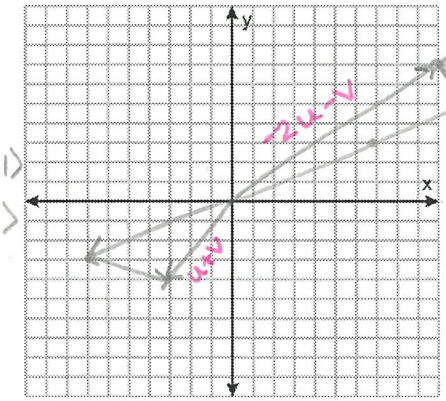
a) $\langle -7,-3\rangle + \langle 4,-1\rangle$

$\langle -3,-4\rangle$

b) $-2\langle -7,-3\rangle - \langle 4,-1\rangle$

$\langle 14,6\rangle + \langle -4,1\rangle$

$\langle 10,7\rangle$



8. $\mathbf{u} = -6\mathbf{j}$

$\mathbf{v} = \mathbf{i} + \mathbf{j}$

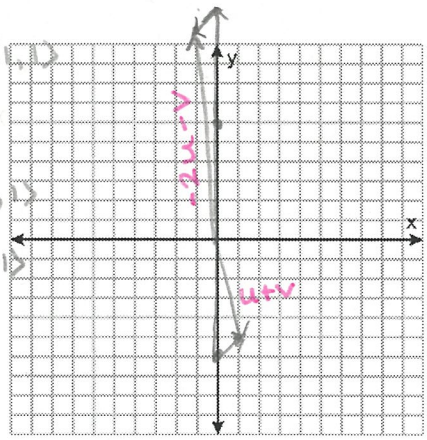
a) $\langle 0,-6\rangle + \langle 1,1\rangle$

$\langle 1,-5\rangle$

b) $-2\langle 0,-6\rangle - \langle 1,1\rangle$

$\langle 0,12\rangle + \langle -1,-1\rangle$

$\langle -1,11\rangle$



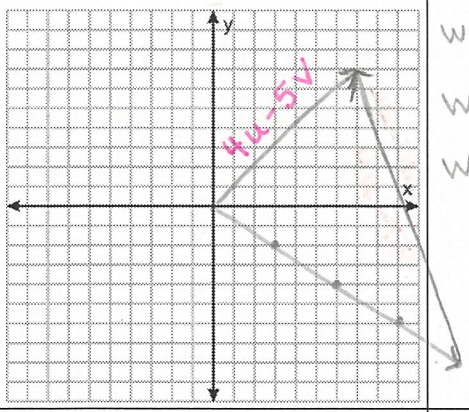
Find the component form of w and sketch the specified vector operations geometrically, where $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$.

9. $\mathbf{w} = 4\mathbf{u} - 5\mathbf{v}$

$w = 12i - 8j - 5i + 15j$

$w = 7i + 7j$

$w = \langle 7, 7 \rangle$

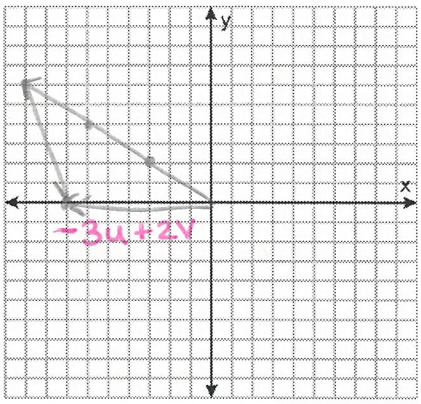


10. $w = -3\mathbf{u} + 2\mathbf{v}$

$w = -9i + 6j + 2i - 6j$

$w = -7i$

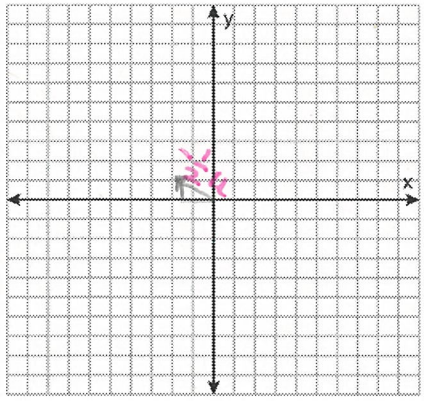
$w = \langle -7, 0 \rangle$



11. $\mathbf{w} = -\frac{1}{2}\mathbf{u}$

$w = -1.5i + j$

$w = \langle -1.5, 1 \rangle$

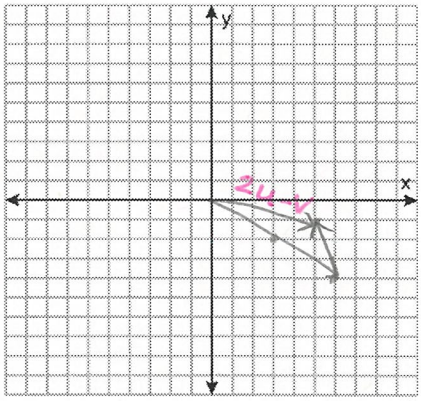


12. $\mathbf{w} = 2\mathbf{u} - \mathbf{v}$

$w = 6i - 4j - i + 3j$

$w = 5i - j$

$w = \langle 5, -1 \rangle$



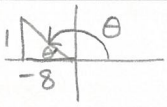
Find the magnitude and direction angle of the vector v.

13. $\mathbf{v} = -8\mathbf{i} + \mathbf{j}$

$\|v\| = \sqrt{64+1}$

$\|v\| = 65$

$\theta = 172.9°$



$\tan\theta = \frac{1}{-8}$

$\tan^{-1}\left(\frac{1}{8}\right) = \theta'$

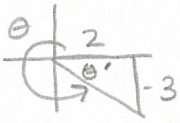
$\theta' = 7.1°$

14. $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

$\|v\| = \sqrt{4+9}$

$\|v\| = \sqrt{13}$

$\theta = 303.7°$



$\tan\theta' = \frac{-3}{2}$

$\theta' = 56.3°$

15. $\mathbf{v} = 7(\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j})$

$v = 7(\frac{1}{2}i + \frac{\sqrt{3}}{2}j)$

$v = \frac{7}{2}i + \frac{7\sqrt{3}}{2}j$

$\|v\| = 7$

$\theta = 60°$

16. $\mathbf{v} = 2(\cos 210°\,\mathbf{i} + \sin 210°\,\mathbf{j})$

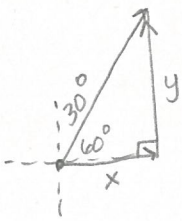
$v = 2(-\frac{\sqrt{3}}{2}i + -\frac{1}{2}j)$

$v = -\sqrt{3}i - j$

$\|v\| = 2$

$\theta = 210°$

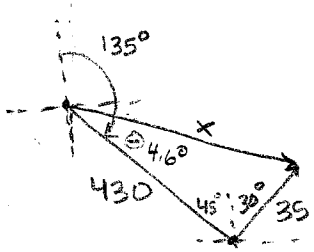
17.



$x = 500\cos 60° = 250$ mph east

$y = 500\sin 60° = 433.01$ mph north

18. An airplane has an airspeed of 430 miles per hour at a bearing of 135°. The wind velocity is 35 miles per hour in the diection of N30°E. Find the resultant speed and direction of the airplane.



$x^2 = 430^2 + 35^2 - 2(430)(35)\cos 75°$

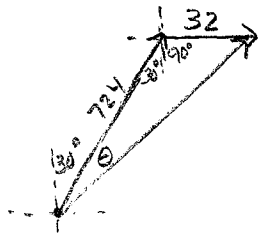
$x = 422.3$

$\frac{\sin\theta}{35} = \frac{\sin 75°}{422.3}$

$\theta = 4.6°$

Bearing 130.4°

19. An airpland has an airspeed of 724 kilometers per hour at a bearing of 30°. The wind velocity is 32 kilometers per hour from the west. Find the resultant speed and direction of the airplane.



$x^2 = 724^2 + 32^2 - 2(724)(32)\cos 120°$

$x = 740.5$

direction 32.14°

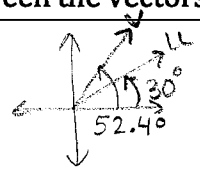
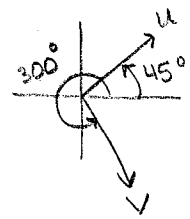
$\frac{\sin\theta}{32} = \frac{\sin 120°}{741.98}$

$\theta = 2.14°$

Find the dot product: $\mathbf{u} \cdot \mathbf{v}$

20. $\mathbf{u} = \langle -1, -7 \rangle$ $\mathbf{v} = \langle 5, -2 \rangle$ $\langle -1,-7\rangle \cdot \langle 5,-2\rangle =$ $-5 + 14 = 9$	21. $\mathbf{u} = \langle 2, -8 \rangle$ $\mathbf{v} = \langle -5, 2 \rangle$ $\langle 2,-8\rangle \cdot \langle -5,2\rangle =$ $-10 - 16 = -26$
22. $\mathbf{u} = -6\mathbf{j}$ $\mathbf{v} = 2\mathbf{i} + \mathbf{j}$ $\langle 0,-6\rangle \cdot \langle 2,1\rangle =$ $0 - 6 = -6$	23. $\mathbf{u} = -2\mathbf{i} - 6\mathbf{j}$ $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$ $\langle -2,-6\rangle \cdot \langle 3,-4\rangle =$ $-6 + 24 = 18$

Find the angle θ (in degrees) between the vectors.

22.4° 24. $\mathbf{u} = \langle 3, \sqrt{3} \rangle$ $\mathbf{v} = \langle 4, 3\sqrt{3} \rangle$ $\frac{21}{\sqrt{12} \cdot \sqrt{43}}$ $\cos\theta = \frac{u \cdot v}{\lVert u \rVert \lVert v \rVert}$  $\theta = 22.4°$	25. $\mathbf{u} = \cos 45°\,\mathbf{i} + \sin 45°\,\mathbf{j}$ $\mathbf{v} = \cos 300°\,\mathbf{i} + \sin 300°\,\mathbf{j}$ $\theta = 255°$ 

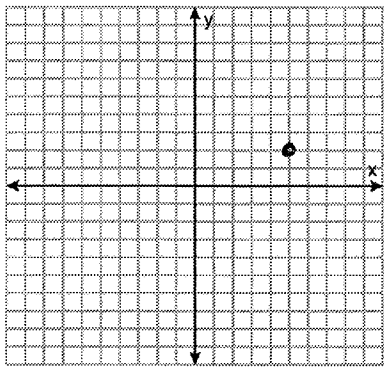
Determine whether u and v are orthogonal. $u \cdot v = 0$

26. $\mathbf{u} = \left\langle \frac{1}{4}, -\frac{1}{2} \right\rangle$ $\mathbf{v} = \langle -2, 4 \rangle$ $u \cdot v = (1/4)(-2) + (-1/2)(4)$ $= -1/2 - 2$ $= -2.5$ NO	27. $\mathbf{u} = -2\mathbf{i} + \mathbf{j}$ $\mathbf{v} = 3\mathbf{i} + 6\mathbf{j}$ $u \cdot v = (-2)(3) + (1)(6)$ $= -6 + 6$ $= 0$ Yes

Plot the complex number and find its absolute value.

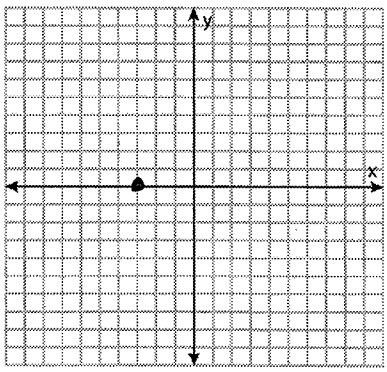
28. $5 + 2i$

$|5+2i| = \sqrt{29}$



29. $-3i$

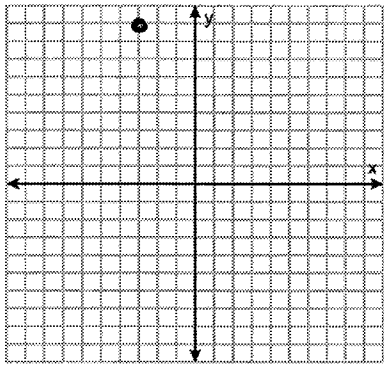
$|-3i| = 3$



30. $-3 + 9i$

$|-3+9i| = 3\sqrt{10}$

≈ 9.5

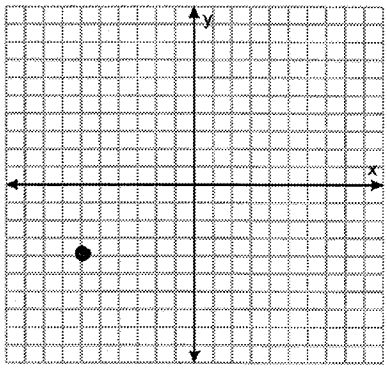


31. $-6 - 4i$

$|-6-4i| = \sqrt{52}$

$= 2\sqrt{13}$

≈ 7.2



Find the sum or difference of the complex numbers in the complex plane.

32. $(4 + 7i) + (-3 - 2i)$

$1 + 5i$

33. $(-5 + 6i) + (+3 + 9i)$

$-2 - 3i$

34. $(10 + 12i) + (+3 + 2i)$

$13 + 14i$

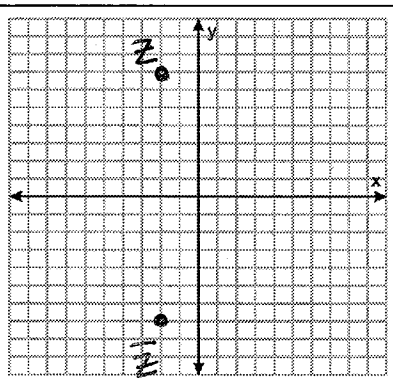
35. $(4 + 7i) + (-4 - 2i)$

$0 + 5i = 5i$

Plot the complex number and its complex conjugate. Write the conjugate as a complex number.

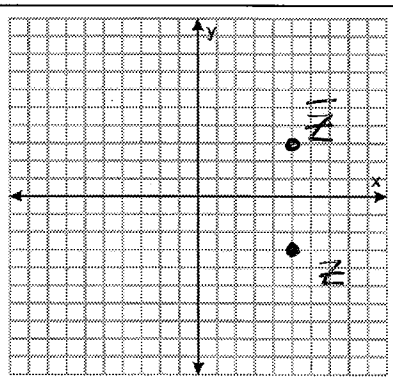
36. $z = -2 + 7i$

$\bar{z} = -2 - 7i$



37. $z = 5 - 3i$

$\bar{z} = 5 + 3i$



Find the midpoint of the line segment joining the points corresponding to the complex numbers.

38. $4 + 8i, -2 - 6i$ $\left(\frac{4+-2}{2}, \frac{8+-6}{2}\right)$

midpoint $(1,1)$

39. $-1 - 7i, 3 - 6i$ $\left(\frac{-1+3}{2}, \frac{-7-6}{2}\right)$

midpoint $(1, 6.5)$

Find the distance between the complex numbers in the complex plane.

40. $1 + i, -5 - 6i$ $(1,1)$ $(-5,-6)$

$d = \sqrt{(1+-5)^2 + (1+-6)^2}$

$d = \sqrt{85}$

41. $-5 + 2i, -1 - 6i$ $(-5,2)$ $(-1,-6)$

$d = \sqrt{(-5+-1)^2 + (2+-6)^2}$

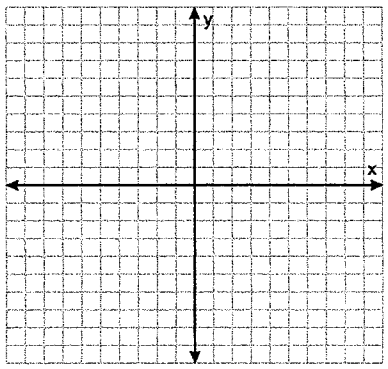
$d = 4\sqrt{5}$

Find the sum or difference of the complex numbers in the complex plane.

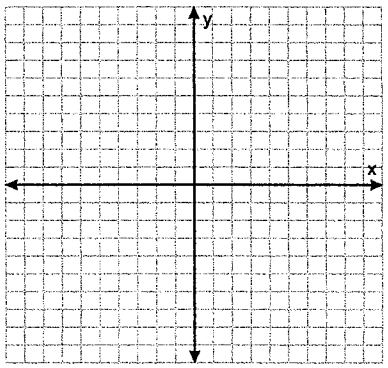
32. $(4+7i)+(-3-2i)$	33. $(-5+6i)-(-3+9i)$
34. $(10+12i)-(-3-2i)$	35. $(4+7i)+(-4-2i)$

Plot the complex number and its complex conjugate. Write the conjugate as a complex number.

36. $z=-2+7i$



37. $z=5-3i$



Find the midpoint of the line segment joining the points corresponding to the complex numbers.

38. $4+8i, -2-6i$	39. $-1-7i, 3-6i$

Find the distance between the complex numbers in the complex plane.

40. $1+i, -5-6i$	41. $-5+2i, -1-6i$

Find the unit vector u in the direction of v. Unit vector: $\mathbf{u}=\frac{\mathbf{v}}{\|\mathbf{v}\|}$

42. $\mathbf{v}=\langle 5,-2\rangle$ $u=\frac{\langle 5,-2\rangle}{\sqrt{29}}=\left\langle \frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}}\right\rangle$	43. $\mathbf{v}=\langle -3,9\rangle$ $u=\frac{\langle -3,9\rangle}{3\sqrt{10}}=\left\langle \frac{-3}{3\sqrt{10}}, \frac{9}{3\sqrt{10}}\right\rangle=\left\langle \frac{-1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right\rangle$

Let $\mathbf{v}=\langle 3,-1\rangle$ and $\mathbf{u}=\langle 0,-2\rangle$. Find the magnitude of each.

44. $\|2\mathbf{u}\|$ $\|2\|\cdot\|u\|=$ $2(2)=4$	45. $\|-4\mathbf{u}\|$ $\|-4\|\cdot\|u\|=$ $4\cdot(2)=8$	46. $\|3\mathbf{v}\|$ $\|3\|\cdot\|v\|=$ $3(\sqrt{10})=3\sqrt{10}$